HEAT TRANSFER BETWEEN A FLUIDIZED BED AND AN IMMERSED CYLINDER

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Inzhenerno-Fizicheskii Zhurnal, Vol. 10, No. 6, pp. 738–743, 1966

UDC 541.182

An analytical formula is given for the heat-transfer coefficient as a function of the frequency of density fluctuations in a fluidized bed and of the radius of curvature of the heat-transfer surface. Calculation coincides with the experimental data.

In examining the heat transfer between a heating surface and a fluidized bed, we shall assume that the heat transfer is accomplished by periodic exchange at the surface of clusters (aggregations) of particles carrying away the heat (when $t_w > t_b$) released by the surface by conduction. Account is also taken of the heat carried away from the heated cluster by the penetrating fluidizing agent.

A cluster of particles is understood to be an aggregation of particles of the fluidized bed and of the moving gas filling the space between them. The cluster is considered to be a homogeneous medium with constant thermophysical properties λ_{eff}, c, ρ over its volume.

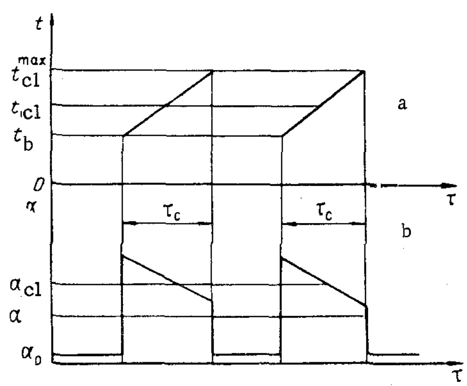

Fig. 1. Quasi-steady process of heat transfer between a fluidized bed and a surface immersed in it: a) periodic heating of clusters at surface; b) variation of α with time.

A motionless cluster of particles is heated at its surface, thus reducing the power of the heat flux; then the heated cluster is thrown off by a gas bubble and is replaced by a new one with temperature t_b. When a bubble is in contact with the surface, the heat-transfer coefficient α and the heat-flux power fall almost to zero, and then increase discontinuously with the arrival at the surface of a new cluster, almost to 2000 $W \cdot m^{-2} \cdot degree^{-1}$ [1].

The frequency of exchange of clusters coincides with the frequency of density fluctuations and varies from 0.5 to 6 cps, depending on the properties of the fluidized material and the fluidizing agent [1, 2].

The unsteady process of heat transfer between a fluidized bed and an immersed surface may be considered as quasi-steady over a long enough time interval, when calculating the time-average heat transfer coefficient and the time-average specific heat flux [3] (Fig. 1b).

The influence of surface curvature on heat transfer has been examined qualitatively by Antonishin and Zabrodskii [4], who based their analysis on a mean temperature over the dwell time of a cluster at the surface, this depending only on the coordinates of the point. Using this concept, we made a quantitative analysis of the process, based on the above ideas about the mechanism of heat transfer in a fluidized bed.

Analytical calculation of the heat-transfer coefficient. If α_{cl} is the time-average coefficient of heat transfer between the surface and a cluster of particles, α_0 is the coefficient of heat transfer between the surface and the gas during passage of a bubble, and f_0 is the fraction of the time for which the surface is in contact with the gas bubble, then the time-average heat transfer coefficient may be represented [3] in the form

$$\alpha = \alpha_0 f_0 + (1 - f_0)\,\alpha_{cl}. \tag{1}$$

Neglecting the quantity $\alpha_0 f_0$ because of the smallness of α_0 in comparison with α_{cl} (f_0 is not more than 0.6) [1], we obtain

$$\alpha = (1 - f_0)\,\alpha_{cl}, \tag{2}$$

where α_{cl} may be written in the form

$$\alpha_{cl} = 1/(R_\lambda + R_\kappa). \tag{3}$$

Here R_λ is the thermal resistance of the mass of a heated cluster; R_K is the additional contact thermal resistance arising in the wall region due to the increase in its porosity [3].

Examining the unsteady cylinder-cluster heat-transfer process, we can write the heat-conduction equation for the cluster in cylindrical coordinates

$$\frac{\partial t_{cl}}{\partial \tau} c\rho = \lambda_{eff}\left(\frac{\partial^2 t_{cl}}{\partial r^2} + \frac{1}{r}\frac{\partial t_{cl}}{\partial r}\right) - q_V. \tag{4}$$

The time-average power q_V of heat drain per unit volume of a cluster is determined from the heat transfer between the particles of the cluster and the gas moving inside it:

$$q_V = \alpha_{gr} F_{gr} (t_{gr} - t_g). \tag{5}$$

To simplify the solution of (4) we replace the cluster heating curve by a straight line (Fig. 1a), i.e.,

$$\frac{\partial t_{cl}}{\partial \tau} \approx \frac{\Delta t}{\Delta \tau} = \frac{t_{cl}^{max} - t_b}{\tau_c} = \frac{2(t_{cl} - t_b)}{\tau_c}, \quad (6)$$

which is quite admissible when the time of contact τ_c of the cluster with the cylinder surface is short enough (in practice $\tau_c = 0.1$–0.5 sec [1, 2]).

Taking into account that α_{gr} is small [6], while the heat capacity of a cluster is large, we may put, as a first approximation,

$$t_{gr} - t_g \approx t_{cl} - t_b = \vartheta. \quad (7)$$

It should be noted that for very fine particles (less than 100 μ), the quantities α_{gr} and q_V are generally insignificant. Substituting in (6)

$$\tau_c = (1 - f_0)/n, \quad (8)$$

we obtain (4) in the form

$$\frac{d^2\vartheta}{dr^2} + \frac{1}{r}\frac{d\vartheta}{dr} - \frac{2c\rho n/(1-f_0) + \alpha_{gr}F_{gr}}{\lambda_{eff}}\vartheta = 0. \quad (9)$$

Solving this equation in boundary conditions of the first kind and designating

$$\frac{2c\rho n/(1-f_0) + \alpha_{gr}F_{gr}}{\lambda_{eff}} = a, \quad (10)$$

we obtain

$$\vartheta = \vartheta_0 K_0(\sqrt{a}\,r)/K_0(\sqrt{a}\,R), \quad (11)$$

where $\vartheta_0 = t_W - t_b$.

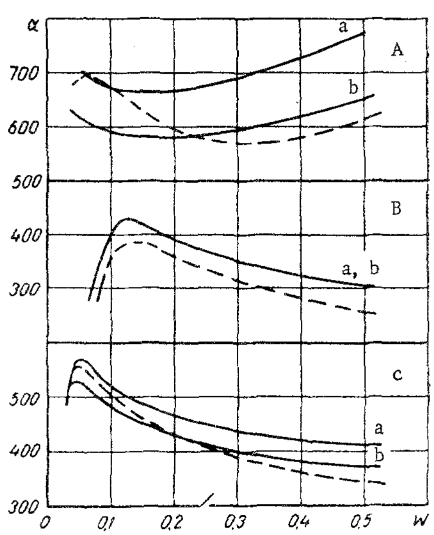

Fig. 2. Comparison of the experimental (solid curves) and calculated (dashed curves) local heat-transfer coefficients α for glass beads of diameter 70 (A), 320 (B), and 152 (C) μ: a) for a point in the bed 365 mm from the distributor; b) 568 mm. The dashed curves—from results of calculation of α at a height of 467 mm. The fluidization velocity W is in m/sec.

Examining (11) together with the equations of the laws of Newton and Fourier

$$q = -\lambda_{eff}\frac{d\vartheta}{dr} = \alpha_\lambda(t_{cyl} - t_b) = \alpha_\lambda\vartheta_0. \quad (12)$$

we obtain an expression for the thermal resistance of the homogeneous mass being heated at the cylindrical surface of the cluster

$$R'_\lambda = \frac{1}{\alpha_\lambda} = \frac{K_0(\sqrt{a}\,r)}{K_1(\sqrt{a}\,R)\,\lambda_{eff}\sqrt{a}}. \quad (13)$$

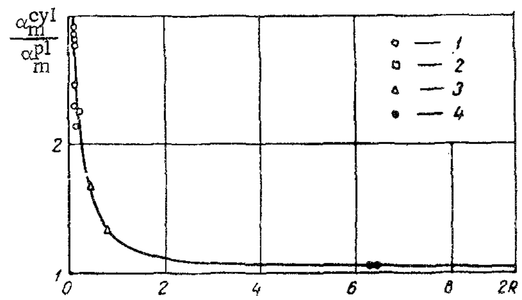

Fig. 3. Influence of cylinder diameter 2R (mm) on the ratio of the maximum heat-transfer coefficient of a cylinder and a plane surface $\alpha_m^{cyl}/\alpha_m^{pl}$: 1) according to [7] (2R = 0.132 mm, glass beads d = 31, 62, 153, 292 μ, H_2, He, CO_2, air); 2) according to [5] (2R = = 0.2 mm, quartz sand a = 140, 198, 215, 357, 515, 745 μ, CO_2, air); 3) according to [8] (2R = 0.4 and 0.5 mm, corundum d = 99 μ, air); 4) according to [1] (2R = 6.35 mm, glass beads, d = = 70, 103, 152, 320 μ, air).

In the case of a plane surface, i.e., as $R \to \infty$ and the ratio of the Bessel functions of imaginary argument of first and zero order $K_0(\sqrt{a}R)/K_1(\sqrt{a}R) \to 1$, combining (13) and (10), we obtain

$$R'_\lambda = \sqrt{\tau_c/2c\rho\lambda_{eff}} \quad (14)$$

in place of the exact value

$$R_\lambda = \frac{1.25}{\beta}\sqrt{\frac{\tau_c}{2c\rho\lambda_{eff}}}, \quad (15)$$

which is obtained from the expression

$$a = \frac{2\beta(1-f_0)}{R_c}\left[1 - \frac{\ln(1 + R_c/R_K)}{R_c/R_K}\right], \quad (16)$$

derived in [3, 8], with $R_K = 0$. Here $R_c = \sqrt{\pi\tau_c/\lambda_{eff}\rho c}$ [3, 8]. The coefficient β is also derived in references [3, 8].

The fact that R_λ is underestimated by $1.25/\beta$ is evidently due to the approximation in transformation (6). Introducing a correction, we have

$$R_\lambda = \frac{1.25}{\beta}\,\frac{K_0(\sqrt{a}R)}{K_1(\sqrt{a}R)\,\lambda_{eff}\sqrt{a}} \quad (17)$$

and finally

$$\alpha = (1 - f_0) \Big/ \left(\frac{1.25\,K_0 (\sqrt{a}R)}{\beta\, K_1 (\sqrt{a}R)\, \lambda_{eff} \sqrt{a}} + R_K \right). \qquad (18)$$

Comparison of the calculated and experimental values. In order to verify the analytical expression obtained, we used the experimental data presented in [1], and the results of their reduction in [3]. The tests were carried out with glass beads of dimension 43, 70, 103, 152, and 320 μ, fluidized with air. The calorimeter was a cylindrical heater of diameter 6.35 mm. Measurements of α were carried out in the fluidization velocity range 0–0.6 m/sec, at distances of 365 and 568 mm from the distributor. Values of n and f_0, and therefore values of α computed from (18), were found for a height of 467 mm above the distributor. The quantity λ_{eff} was taken from the data of measurements of the authors of [1]. The tests [1] show that the maximum value of α is reached at a frequency somewhat greater than that of bubble passage, i.e., the frequency of total exchange of clusters. Evidently bubbles which pass alongside a cluster displace the layer of particles situated on the surface, without disturbing the position of the cluster as a whole [3]. In our calculations we used the value of precisely this frequency, which can also be assumed as the calculation value of the frequency of exchange of clusters [1, 3, 8]. The experimental and calculated curves are shown in Fig. 2. Analysis of the combined curves allows the conclusion to be drawn that in the whole range of fluidization velocities presented, the possible error in calculation of the heat-transfer coefficient is small. The discrepancy of the curves at large fluidization velocities may be explained by our neglect of heat exchange with the gas of the bubble, which becomes noticeable in this case.

Analysis of the results obtained. To calculate the heat-transfer coefficient α it is necessary to know the quantities λ_{eff} and a.

The coefficient appearing in (10) of heat transfer between particles of a cluster and the fluidizing gas may be found from the graphs given in [6], or determined from the formula also given there.

In general the quantity $\alpha_{gr}F_{gr}$ for fluidization of particles less than 100 μ is negligibly small in comparison with the quantity $2c\rho n/(1 - f_0)$ entering into (10).

For a small temperature drop ϑ_0 and weak dependence of the thermophysical properties of the fluidized bed on temperature, the calculation may be done with the constants at the bed temperature.

The fraction f_0 of the dwell time of the calorimeter in the gas phase (bubble) is equal on the average to the fraction of the volume occupied by bubbles at a given place in the bed: $f_0 = 1 - \rho_l/\rho$.

The contact thermal resistance R_K of the zone at the wall may be calculated from the formula given in [3]

$$R_K \quad d \Big/ \pi\lambda_g \left[\ln \frac{\lambda_M}{\lambda_g k} - 1 \right]. \qquad (19)$$

It is true that this expression does not contain the diameter of the cylindrical calorimeter, which undoubtedly has an influence on R_K when the calorimeter diameter is commensurate with the particle diameter.

By analyzing (17) and (18), we can shed light on the nature of the dependence of α on the cylinder diameter 2R. In the case of the plane surface

$$R_\lambda^{pl} = 1.25/\beta\lambda_{eff}\sqrt{a}. \qquad (20)$$

Calculation shows that for a cylinder diameter of 5 mm, the deviation of α from the value computed for a plane surface, α_{pl}, is not more than 5%. With decrease of cylinder diameter to less than 3–4 mm, α increases sharply, as is clearly demonstrated in Fig. 3. Here the values of α_m^{cyl} and α_m^{pl} are calculated for fluidized beds of glass beads of dimensions 70, 152, and 320 μ, using values of f_0 and n taken from [1]. The curves constructed from the results of these calculations practically coincide for the fluidized beds of particles of different diameters.

There is satisfactory agreement between the curve shown and experimental points obtained by various authors. The missing values of maximum heat-transfer coefficient α_m^{pl} in the plane case, for bed conditions given in [7] and in [1] have been calculated from the Varygin formula given below.

The empirical formulas obtained by Varygin [5] for a solid body (plane surface case)

$$\mathrm{Nu}_m = 0.86\,\mathrm{Ar}^{0.2} \qquad (21)$$

and for a wire of diameter 0.2 mm

$$\mathrm{Nu}_m = 2\,\mathrm{Ar}^{0.2} \qquad (22)$$

give quite accurate confirmation of calculations according to our Eqs. (18) and (20) when Ar $\geq$ 28 (formula (22) was verified in the range of Ar from 246 to 12 800 [5]).

It is evident that (21) and (22) may be generalized in the form

$$\mathrm{Nu}_m = K\,\mathrm{Ar}^{0.2}. \qquad (23)$$

The values of K calculated by us, using (18)–(20) for cylinders of various diameters, are:

2R, mm	K
0.1	2.3
0.2	1.950
0.4	1.5
0.8	1.09
1	1.030
2	0.946
4	0.910
6	0.895
8	0.890
10	0.880
∞	0.860

Thus, knowing the hydrodynamics of the fluidized bed (n and f_0), we may calculate the coefficient of heat transfer α between the bed and a cylinder immersed in it. It should be noted, however, that the coefficients

K presented still require careful experimental verification.

NOTATION

t_ω, t_b, t_{gr}, t_g – temperatures of the calorimeter surface, the fluidized bed, the particles of a cluster, and of the fluidizing agent (gas), °C; t_{cl}^{max}, t_{cl} – maximum and time-average temperatures of a heated cluster at distance r from the calorimeter center, °C; τ, τ_c – time (variable) and time of contact of a cluster with the surface, sec; f_0 – fraction of the time during which the surface is in contact with the gas bubble; n – frequency of exchange of clusters, sec^{-1}; λ_{eff}, λ_g, λ_M – thermal conductivities: effective for a cluster, of the gas, of the particle material, $W \cdot m^{-1} \cdot degree^{-1}$; R_λ and R_K – time-average thermal resistance of a cluster of particles being heated, and contact thermal resistance, $m^2 \cdot degree \cdot W^{-1}$; α, α_m, α_{cl}, α_0, α_{gr}, α_λ – coefficient of heat transfer between the calorimeter surface and the fluidized bed, maximum and time-average between the surface and a cluster of particles, allowing for R_K, between the surface and the gas of the bubble, between the particles of a cluster and the gas, between the surface and the homogeneous mass of a cluster (without allowing for R_K), $W \cdot m^{-2} \cdot degree^{-1}$; R_λ^{pl}, α_m^{pl}, α_m^{cyl} – values of R_λ and α_m, respectively, for a plane and a cylinder; c – specific heat (mass) of a cluster, time-average, $j \cdot kg^{-1} \cdot degree^{-1}$; ρ_M, ρ, ρ_g, ρ_l – density of the material of the particles, of the cluster, of the gas, and the mean over a section of the bed, $kg \cdot m^{-3}$; F_{gr} – surface area of particles in unit volume, m^{-1}; r, R – variable radius and calorimeter radius, m; d – diameter of particles of the fluidized bed, m; W – fluidization velocity, $m \cdot sec^{-1}$; ν – kinematic viscosity of the gas, $m^2 \cdot sec^{-1}$; g – acceleration due to gravity, $m \cdot sec^{-2}$; $Ar = (gd^3/\nu^2)(\rho_M \rho_g/\rho_g)$ – Archimedes number; $Nu_m = \alpha_m d/\lambda_g$ – Nusselt number for α_m; $K_0(\sqrt{a}R)$, $K_1(\sqrt{a}R)$ – modified Bessel functions of the first kind of order zero and one in the argument $(\sqrt{a}R)$, 1/k – fraction of the particle half-thickness with nonzero temperature gradients [3].

REFERENCES

1. H. S. Mickley, D. F. Fairbanks, and R. D. Hawthorn, Paper No. 8 at the Heat Transfer Conference, New York, August 14–17, 1960.
2. H. S. Mickley and D. F. Fairbanks, A. I. Ch. E. Journ., 1, September, 1955.
3. A. P. Baskakov, IFZh, **6**, no. 11, 1963.
4. N. V. Antonishin and S. S. Zabrodskii, IFZh, **6**, no. 11, 1963.
5. N. N. Varygin, Tr. MIKhM, **26**, 1964.
6. Daizo Kunii and J. M. Smith, A. I. Ch. E. Journ., **7**, no. 1, 29–36, 1961.
7. A. Jacob and G. L. Osberg, The Canad. Journ. of Chem., no. 1, June 5–9, 1957.
8. A. P. Baskakov, V. A. Antifeev, and G. K. Malikov, Proc. of All-Union Conference on Heat and Mass Transfer [in Russian], Minsk, May, 1964.

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